ADDITIONAL MATHEMATICS

Paper 0606/11
Paper 11

Key messages

Questions should be read carefully to avoid misunderstandings of the situation being described, such as in **Questions 1, 9(b)** and **10(b)**. Reading the question carefully will also ensure that candidates meet all the demands of the question, including the writing of final answers in exact form where required. Special care needs to be taken when manipulating expressions both arithmetically and algebraically. Correct use of brackets and notation is also essential.

General comments

The paper achieved a wide range of marks, but it was evident that some candidates were ill-prepared for the examination rather than constrained by lack of time, as shown by some low scoring scripts. Candidates sensibly used additional sheets when working space was an issue. When using additional sheets it is important that they number the question. It is also helpful to the examiner if they reference the additional sheet in the main question paper. It is important that candidates set out their work in a clear and concise manner so that solutions can be followed easily.

Comments on specific questions

Question 1

Very few completely correct solutions were seen. The question demanded that the expression was to be given in terms of factors with integer coefficients. However, many candidates gave factors with non-integer coefficients such as $\left(x-\frac{1}{2}\right)$. Most candidates realised that a factor of 3 was implied in the expression. In spite of being informed of the required answer form, some candidates still multiplied out their answer.

Question 2

(a) Most candidates wrote the given expression in terms of powers of 2 and showed sufficient detail to obtain the required result.

(b) Most candidates took note of the word 'Hence' and made use of the given result for **part (a)** and solved the simultaneous equations correctly. However, some candidates gave the positive solutions only. Almost all candidates gave their answer in exact form as required.

Question 3

Very few completely correct solutions were seen. It was essential that candidates find the value of b first and then use the given equation of the curve, together with the coordinates of two of the points that could be read from the graph, to find the values of a and c. Many candidates did find the correct value of b but were unsuccessful finding the other constants, with a common misconception that the value of a was 6.

Question 4

It was necessary that candidates recognised that the answer contained a form of $\ln(4x+3)$. While many candidates did and went on to substitute the given limits and apply the laws of logarithms correctly, some made no use of logarithms in spite of the given result being a logarithmic term.

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Question 5

(a) Most candidates attempted to write the term involving x^3 as ${}^{15}C_3\left(1\right)^{12}\left(k\right)^3$ or equivalent. Evaluation to obtain the value of *k* after the term was equated to –29 120 was usually successful. Errors and less successful methods included using *k* rather than k^3, and attempting to expand $\left(1+kx\right)^{15}$ term by term in order to find the required term.

(b) It was evident that candidates who were successful in **part (a)** were more likely to be successful in **part (b)**. Many identified the required term correctly and evaluated it, although there were some errors caused by misuse of the negative term.

Question 6

Most candidates realised that differentiation was needed and correctly found the value of *b*. Most candidates recognised the notation and did not confuse it with that for an inverse function. Correct use of both the remainder and factor theorems together with the solution of the resulting simultaneous equations often resulted in a completely correct solution. Some candidates made arithmetic slips in their work and obtained non-integer values for *a* and/or *c*. As candidates are told that all 3 unknowns are integers, once a non-integer response is obtained, a careful check of working should be made to identify what is usually a minor error and correct it.

Question 7

(a) Few completely correct solutions were seen. Many candidates made use of a straight-line equation and correctly obtained the value of the gradient. However, incorrect substitution of the given value for e^{3y} and x^3 usually resulted with no further marks being available. The same applied to candidates who attempted the solution of two simultaneous equations with the given coordinates. Some candidates, having obtained a correct straight-line equation, did not complete their solution by relating it to the form $e^{5y} = mx^3 + c$ and writing their final answer in the form of *y* in terms of *x*.

(b) Few correct solutions were seen as many candidates were making use of equations which were not in the correct form. It was essential that candidates be considering the fact that a logarithmic term has to be positive.

Question 8

This was an unstructured question. Candidates were expected to integrate twice with respect to *x* making use of the 'reverse chain rule'. It was essential that a constant of integration was included after both sets of integration and the given conditions used to evaluate these constants of integration. Many candidates obtained full marks, but errors usually included omission of one or both of the constants of integration, and coefficient errors when integrating.

Question 9

(a) Most candidates were able to gain some marks for the differentiation of a quotient. Errors usually occurred with the differentiation of e^{-3x+2}. Some candidates found it difficult to obtain their derivative in the form required, highlighting the need to be familiar with standard derivatives.

(b) It was essential that candidates read the question carefully. They were expected to use their response to **part (a)** as highlighted by the use of the word 'Hence'. They were also expected to show that there was only one stationary point on the curve. It was expected that candidates discount the result $e^{-3x+2} = 0$ by stating that there was no solution to this equation. Few candidates did this. Many candidates did obtain the correct *x*-coordinate of the stationary point but occasionally omitted finding the *y*-coordinate and occasionally giving the *y*-coordinate in decimal form rather than exact form.

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Question 10

(a) Most candidates were able to write down the correct equations for the given terms of the geometric progression. Some candidates had difficulty in solving these equations simultaneously to obtain the common ratio and first term. Some candidates found one of the unknowns and omitted the other. There were a few candidates who used the sum of a geometric series rather than the correct general term formula.

(b) Candidates found this question part demanding. Very few completely correct solutions were seen. Most candidates were able to identify the correct common ratio, fewer went on to state the correct conditions for a sum to infinity to exist. Very few candidates found the correct critical values and hence the correct inequalities.

Question 11

Very few completely correct solutions were seen as candidates found this question particularly challenging. Many candidates managed to obtain 3 marks by finding the y-coordinates of the points A and B and then the area of the trapezium underneath the shaded area. There were some attempts at integration, but these were seldom completely successful. The main problem was keeping working in exact form.

Question 12

(a) This question part was completed with varying levels of success. To make any progress, candidates needed to make use of a correct trigonometric identity and obtain a 3-term quadratic equation in terms of either $\cot\theta$ or $\tan\theta$. Candidates also had to ensure that, somewhere in their solution, cotangent was dealt with correctly. Candidates who progressed to this stage often only gave positive solutions rather than considering any negative solutions.

(b) Few completely correct full solutions were seen. Most candidates were able to use the correct order of operations to obtain values of $2\phi + 1.5$. Unfortunately, some candidates gave insufficient solutions to obtain the required angles in the given range.

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ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Questions should be read carefully to avoid misunderstandings of the situation being described, such as in **Questions 1, 8** and **10(b)**. Reading the question carefully will also ensure that candidates meet all the demands of the question, including the writing of final answers in exact form. Special care needs to be taken when manipulating expressions both arithmetically and algebraically. Correct use of brackets and notation is also essential.

General comments

The paper achieved a wide range of marks, but it was evident that some candidates were ill-prepared for the examination rather than constrained by lack of time, as shown by low scoring scripts. It was pleasing to see that candidates used additional sheets when working space was an issue. When using additional sheets it is important that they number the question. It is also helpful to the examiner if they reference the additional sheet in the main question paper. It is also important that candidates set out their work in a clear and concise manner so that solutions can be followed easily.

Comments on specific questions

Question 1

(a) Very few completely correct solutions were seen. It was essential that candidates find the value of b first and then use the given equation of the curve together with the coordinates of the two given points to find the values of a and c. Many candidates did find the correct value of b but were unsuccessful finding the other constants. Too many candidates assumed that b was equal to $\frac{2\pi}{3}$, the period of the curve. Too many candidates also assumed that the coordinates given were the maximum and minimum points on the curve and made their calculations erroneously. In addition many candidates assumed they were dealing with a straight line and found the gradient between the two given points.

The errors mentioned above highlight the need for candidates to read the question carefully.

(b) Many candidates did not attempt this part. For those that did, follow through marks were available provided that $b \neq \frac{2\pi}{3}$ and some fully correct solutions were seen. It was intended that candidates observe that a minimum value occurred when $\cos bx = -1$. Finding the minimum value was then a straightforward substitution into their equation from **part (a)** and the value of x, a solution of $\cos bx = -1$. The most successful candidates, however, were those who made use of calculus.

Question 2

(a) There were quite a few candidates who made no attempt at using calculus to find the coordinates of the stationary points, with many finding the intercepts with the x-axis. Of the candidates who did use calculus, there appeared to be an even split between those who chose to use the product formula for differentiation and those who expanded the function before differentiation. Arithmetic and algebraic slips in working were quite common, resulting in incorrect stationary points. It should

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also be noted that many candidates did not find the y-coordinates of the stationary points, again providing evidence of the question not being read carefully.

(b) Many candidates did not attempt to sketch the graph, and some that did sketched quadratic curves, straight lines or non-standard curves. Many candidates sketched a correct shape, but with an incorrect position. Despite the demand to state the intercepts with the axes, some candidates omitted the values completely whilst others omitted the $x = 2.5$ value.

(c) Very few correct solutions were seen. Many candidates did not attempt this part of the question having performed poorly in the earlier parts. Other candidates appeared not to appreciate the use of the word 'Hence', indicating the use of previous work to determine the values of k. In this case, candidates should have been guided by the amount of working space and mark allocation which suggested that very little extra work needed to be done.

Question 3

(a) Many completely correct solutions were seen with most candidates showing sufficient detail and accuracy to obtain both the available marks.

(b) Again, many completely correct solutions were seen. Errors that were made were often due to using an incorrect value for the radius.

(c) Fewer correct responses were seen. Most candidates were able to make a correct use of the sector area formula. Most errors were often made in the calculation of the area of the kite.

Question 4

(a) (i) Many correct solutions were seen. Candidates should take care when writing their answer. A solution involving an inequality and a did not obtain the mark, as a can only take one value. A solution involving a correct inequality and x was acceptable.

(ii) Very few correct solutions were seen. Candidates are advised to ensure that they use correct notation if they refer to the function in their answer, e.g. an answer of $x \in \mathbb{R}$ would not gain the mark. Correct inequalities were also acceptable.

(iii) Most candidates were able to obtain the correct inverse function although there were quite a few who appeared not to realise the relationship between logarithmic and exponential functions. This question part was another example of candidates not ensuring that they had met the full demands of the question, with many candidates making no attempt at writing down the range and domain of the inverse function. Of those that did, some made notation errors, the most common being $\mathrm{f} > \frac{2}{3}$, rather than the correct $\mathrm{f}^{-1} > \frac{2}{3}$.

(iv) Very few candidates obtained full marks for this question part. Many candidates scored no marks, being unfamiliar with the form of the graph of a logarithmic function. Errors included omitting to state the intercepts with the axes, sketching the curve(s) in the first quadrant only, incorrect shapes, incorrect intercepts and no points of intersection.

(b) Most candidates realised how to deal with the composite function but often made errors with brackets which meant that no meaningful progress could be made. Of the candidates that had a correct initial relationship, most exhibited poor algebraic manipulation, usually involving the square root, which resulted in incorrect subsequent statements.

Question 5

(a) Many completely correct solutions were seen with most candidates showing sufficient detail and accuracy to obtain the available mark. Candidates should be guided by the mark allocation which implied that only a couple of lines of solution was expected. Most candidates chose to change the left-hand side of the given expression to terms in sine and cosine. Others chose to make use of the identity $\tan^2\theta + 1 = 0$. There were also some lengthy solutions which eventually achieved the given result correctly.

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(b) Most candidates were familiar with the required derivative.

(c) Many correct solutions were seen with candidates using the results from **parts (a)** and **(b)** correctly. There were sign errors with the integration of $\sin\theta$ and some candidates did not give their final answer in exact form as required.

Question 6

(a) Many completely correct solutions were seen showing that candidates had a good understanding of the binomial expansion and simplification of the resulting terms. Some candidates did misunderstand the phrase 'descending powers of *x*' giving an attempt at the last three terms of the expansion.

(b) Another question part that showed candidates had a good understanding of the binomial expansion. Many candidates were able to identify the required term by observation, while others calculated which term was required. A few candidates attempted a full expansion and stopped when they reached the constant term. It was pleasing to see that many candidates simplified their terms correctly and obtained the required answer.

Question 7

Most candidates were able to gain some marks for the differentiation of a quotient. Errors usually occurred with the differentiation of $\ln\left(3x^2-1\right)$. Some candidates were not familiar with rates of change and thought that they had to find a value for *h*. Most realised that a substitution of $x=1$ was needed, but poor simplification meant that a correct value for $\frac{dy}{dx}$ when $x=1$, was not obtained. Fewer candidates used a correct approach to find the required rate of change and some gave their final answer as a small change. Again, there were candidates who did not appreciate that an exact answer was required and resorted to their calculator and decimals.

Question 8

An unstructured question, the aim of which some candidates misunderstood. It was essential that a correct expression for $\frac{dy}{dx}$ be obtained. However, some candidates made errors with the powers of $(2x+5)$, sometimes omitting them. Incorrect simplification when $x=2$ was substituted was quite common. It was important from the start of the question for candidates to realise that the answer was required in exact form, so no use of a calculator was expected. Unfortunately, too many candidates resorted to the use of their calculator. Many candidates identified a correct approach by finding the equation of the tangent, but poor algebra often led to incorrect intercepts. Some candidates did not find the equation of the tangent and appeared to find the midpoint of random points. This question was another one where it was of paramount importance to read the question thoroughly before making any attempt at solution.

Question 9

Another unstructured question which many candidates were able to make a reasonable attempt at a solution. Most realised that they had to find the coordinates of the point of intersection of the straight line and the curve. This was done successfully by many with a few making arithmetic and algebraic slips which led to an incorrect quadratic equation. Finding the shaded area meant considering the appropriate area under the curve and subtracting the area of the appropriate trapezium. Some chose to use integration using subtraction of the straight-line equation and the curve equation. This was equally acceptable. Problems occurred with arithmetic slips, but most realised that the integral $\ln(2x+1)$ was involved. There was evidence of poor use of brackets by some candidates when dealing with $\ln(2x+1)$ and the substitution of limits. In some cases, omission of brackets meant obtaining an incorrect evaluation and hence an incorrect solution. Many correct solutions were seen, although it should be noted that too many final answers of $2\ln 2-\frac{5}{8}$ were seen, rather than the required $\ln 4-\frac{5}{8}$. This again highlights the need to read the question carefully and ensure that the final answer is given in the required form.

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Question 10

(a) This question part was quite demanding as it involved dealing with trigonometry as well as an arithmetic progression. Too many candidates made errors with identifying the common difference and/or the application of the correct formula for the sum of an arithmetic progression. Poor simplification led to an incorrect trigonometric equation. Some candidates were able to achieve a correct trigonometric equation and make a reasonable attempt at solution. It was evident that candidates were more comfortable dealing with positive angles than negative angles as many solutions just contained the positive solutions.

(b) This question part was another example of candidates not reading the question carefully. Too many candidates saw the phrase ‘sum to infinity’ and having found a correct common ratio, immediately formed a sum to infinity with it. Unless a correct inequality for the common ratio was stated and used, candidates were unable to obtain any further marks. For those candidates that did use a correct inequality, critical values were given rather than the ranges of values that were required. Very few candidates achieved full marks for this question part.

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ADDITIONAL MATHEMATICS

Paper 0606/13
Paper 13

Key messages

Candidates should be reminded to read each question carefully and to pay attention to the required form of an answer. It would be advantageous for centres to draw candidates' attention to the instructions on the front of the examination paper, especially those related to accuracy. Many answers were rounded to an incorrect degree of precision, and in some cases, premature rounding during intermediate steps led to incorrect conclusions resulting in a loss of marks. When sketching a graph, candidates should remember to state clearly the intercepts of the x and y axes.

Some candidates tended to reject negative results in contexts where they were mathematically valid. A more thorough understanding and familiarity with such cases would help improve their overall performance in these situations.

General comments

There appeared to be minimal issues with timing during the examination, and candidates who omitted questions seemed to do so primarily due to a lack of understanding rather than time constraints.

Candidates could have improved their performance by exercising greater care, particularly when manipulating expressions and equations. Sign errors were common, especially in **Questions 5** and **8**. Additionally, more attention to detail when handling and writing numerical values would have been beneficial, as many candidates misread their own figures.

A significant number of candidates displayed an incomplete understanding of logarithmic theory, particularly in questions involving expressions of logarithms. This lack of understanding hindered their progress in **Questions 4**, **6**, and **10(a)**. It is evident that many would benefit from further practice in this area.

Comments on specific questions

Question 1

(a) Most candidates correctly expanded the brackets and then differentiated the resulting expression to find the coordinates of the turning points. A significant minority completed the square correctly to achieve the same result. Only a very few candidates used a symmetry argument to find the turning point. Some candidates stopped after finding the x-coordinate of the turning point, without proceeding to find the corresponding y-coordinate. Occasionally, candidates expanded the brackets incorrectly before differentiating, earning a method mark for the correct approach, but not for full completion. A small number of candidates made little or no progress at all.

(b) Some candidates produced very good work, correctly identifying and plotting the turning points and maxima. However, many candidates lost a mark, typically due to one of the following reasons:

- The maximum was plotted at the point (0, 12) which was not the correct coordinate for the maximum.
- The axes were not labelled correctly, which led to confusion in identifying the correct location of the turning points or maximum.
- In some cases, the curvature of the graph was incorrect, particularly at the cusps or on either side of the cusps.

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(c) Many candidates provided correct answers, building on their correct solutions to **parts (a)** and **(b)**. These candidates demonstrated a clear understanding of the problem and were able to progress logically to the final answer. However, a few candidates introduced an error by incorrectly adding $k = 0$, which invalidated their solution. A number of candidates, despite previous correct steps, started again and gave answers that indicated a lack of understanding of what was being asked, leading to incorrect solutions that were disconnected from the earlier parts of the question.

Question 2

For the first mark, a valid graph must be shaped correctly, with endpoints located at (–360, 4) and (360, 4). It should feature a minimum point in the 3rd quadrant and a maximum point in the 1st quadrant. Once this initial mark is secured, additional marks can be awarded for accurately placing the maximum and minimum points.

While many candidates produced correct graphs, the intercept at (0, 4) was frequently omitted. Candidates who calculated the *y*-coordinates for *x* values of –360, –180, 0, 180, and 360 typically generated reasonable answers, but their graphs sometimes lacked smooth curves. Another common mistake involved misrepresenting the sine function by using a wavelength of 180° instead of the required factor of 2, resulting in incorrect stretching. Occasionally, candidates sketched an inverted curve. Marks were also withheld if the turning points deviated too far from their accurate positions. It is recommended that candidates engage in more practice with curve sketching in order to enhance their skills in accurately placing the key features of their graphs.

Question 3

Some candidates lost the first mark by incorrectly setting the quadratic expression $f(x) < 0$ instead of solving the equation $f(x) = 0$, or by making a sign error, typically when dealing with the coefficient 2. These candidates did not correctly identify the critical values, which resulted in the loss of the first mark. Candidates who earned the first mark typically proceeded to correctly finding the two critical values of *x*, where the quadratic expression equals zero.

After finding the critical values, many candidates made no further progress in solving the inequality. However, most of the more proficient candidates correctly proceeded by writing out the inequality in its appropriate form and solving it correctly.

Question 4

(a) The expression involved the need to convert the number 3 into a logarithm to base 2, which many candidates did not do. While many candidates correctly performed this step, others left 8 as 2^3, which was not accepted for the final mark. Next, the term $4\log_2 a$ needed to be rewritten as $\log_2 a^4$ before it could be combined with the other terms, producing the expression $\log_2\left(\frac{a^4}{b}\right)$. However, some candidates misused logarithmic properties and attempted to express the result as $\frac{\log_2 a^4}{\log_2 b}$, which earned no credit. A few candidates attempted to use logarithms with bases other than 2 but these attempts were unsuccessful.

(b) The first step needed to be to change the base on one of the logarithms thus producing either $\lg x = \frac{4}{\lg x}$ or $\frac{1}{\log_x 10} = 4\log_x 10$ and whilst doing so, some did not always seem to appreciate that $\log_{10} x$ and lg x are the same thing. Once this change had been made, the next step should have produced $(\lg x)^2 = 4$ or $(\log_x 10)^2 = \frac{1}{4}$, although many wrote $\lg x^2 = 4$ which is incorrect, even though it did produce the required answers. Quite often lg *x* or $\log_x 10$ was sensibly replaced in the equation with a dummy variable which did prevent the above error. Some candidates used *x* as their dummy variable which is not advised as some were then unclear about their own working. Candidates should always use a letter that does not appear in the question and clearly state what it stands for. Thereafter it was straightforward to produce $\lg x = \pm 2$ or $\log_x 10 = \pm 0.5$ and thus

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produce the answers of 100 and 0.01. However, many candidates rejected the negative solution or did not write it down and consequently could not be awarded full marks.

Question 5

(a) Those candidates who attempted long division were generally the least successful in solving this part. This method, while valid, was often more complex and prone to errors. In contrast, the most successful candidates rearranged both equations to express *c* in terms of the other variables and then either equated the two expressions for *c* or substituted one expression into the other. This approach allowed for a more direct solution, and the candidates who followed it tended to perform better. A common error were sign errors in the terms involving *b*, particularly when manipulating terms during the process. Some candidates also forgot to equate $p(-2)$ meaning an important step in the solution was missing.

Some candidates performed intermediate steps, such as evaluating the two equations, but did not clearly show the methods they used afterwards. As this was a 'Show that…' question, this lack of clarity in the process often resulted in lost marks for method.

(b) Many candidates were able to correctly solve this part. However, some candidates made the mistake of forgetting to evaluate *c* after deriving expressions for *a* and *b*. This occurred because the two given equations were expressed in terms of *a* and *b*, and candidates needed to substitute the values of *a* and *b* back into the equation to find the value of *c*.

A few candidates did not recognise that the equation $7a - 9b = 39$ was the second equation to use. This equation could be directly substituted or manipulated to find values for *a* and *b* efficiently. However, some candidates either overlooked this equation or chose a more complex equation, leading to unnecessary complications in their solution process.

Question 6

(a) Candidates were required to calculate the values of $\ln y$ and x^2 from the given table and use those values to plot five points on a graph. These points were to be connected with a straight line. While many candidates successfully completed this task, some encountered difficulties with the scaling. Specifically, the values 2.25 and 6.25 on the horizontal scale were frequently plotted incorrectly. Additionally, the value $\ln y = 8.18$ was often recorded as 8.1. Nearly all candidates drew a straight line through their points, which was helpful for those who checked the intercept on the vertical axis, which would later be used in **part (b)**.

(b) Candidates needed to correctly transform the given equation $y = Ab^{x^2}$ into the linear form, $\ln y = \ln A + x^2 \ln b$ However, some candidates mistakenly placed x^2 as a coefficient of $\ln A$, while others attempted to use logarithms with base 10, which was not successful. After this transformation, candidates were expected to use their graph to calculate the gradient, which would then be equated to $\ln b$ providing the value of b. This approach was often successful, although some candidates calculated b first and then used it to find *A*, which was prone to error if b was not accurately determined. Some candidates ignored the instruction to use their graph and used the original equation $y = Ab^{x^2}$, substituting values from the given table. This was condoned on this occasion. When calculated correctly, this method provided acceptable results. However, a single error in the working usually led to both results being incorrect, resulting in zero marks. Rarely, this approach was accompanied by an equation for $\ln y$, meaning that not all marks were awarded.

(c) In order to find the value of *x* when $y = 200$, candidates need to find ln 200 and locate that value on the graph. They then needed to read off the corresponding value for x^2, square root and round to 2 significant figures as requested. A common error was to find ln 200 but to take this value from the horizontal axis rather than the vertical axis. Some also made the error of assuming that their value from the graph was *x* rather than x^2. Many candidates did not round their answer as requested.

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Some used their result from **part (b)** which was acceptable for the method mark as long as they correctly reached a result for x^2, although using the original equation often produced $y = 7 \times 2^{x^2}$ which was sometimes turned into the incorrect form of $y = 14^{x^2}$.

Question 7

(a) Most candidates were able to correctly apply the product rule to differentiate the given expression. However, many encountered difficulties in manipulating the terms after applying the product rule. A common error was to subtract rather than add the two terms when applying the product rule. Some candidates did not know the derivative of $\ln x$.

(b) Many candidates struggled with simplification in **part (a)**, which hindered their progress in solving this part. Some candidates did not recognise the connection between the two parts of the question, despite the use of the word 'Hence…', which affected their ability to complete the problem correctly. Despite this, many attempts were well-executed overall. Common minor errors included leaving $\ln 256$ as b and arithmetic errors when adding the fractions.

Question 8

(a) Most candidates provided fully correct solutions or nearly correct ones. However, some weaker candidates struggled with manipulating the given equations to obtain a quadratic in x. These candidates either had difficulty with the algebraic manipulation or made careless errors, which resulted in a loss of marks. Common errors included; forgetting to evaluate the y-coordinates, not finding a midpoint, differentiating $y = 2x + 1$ to find the gradient or forgetting to find and use the perpendicular gradient.

(b) Few candidates knew how to approach this part of the question effectively. Those who used a vector approach were often successful, as this method provided a straightforward way to handle the geometric relationships involved. However, many candidates attempted to solve the problem using Pythagoras' theorem, but this often led to complicated algebraic expressions. While Pythagoras' theorem could be relevant in some situations, its application here was not always straightforward. Candidates who used this approach frequently ended up with either no meaningful result or an incorrect final result due to errors in algebraic manipulation or an incorrect setup of the problem.

Question 9

Those candidates with a solid grasp of vector algebra were typically able to solve the problem correctly. However, a few candidates made sign errors in **parts (a)** or **(b)**, which led to incorrect results and thus a loss of marks.

While many candidates made good progress through **parts (a)** to **(d)**, some made no progress in the final part due to incorrectly equating the wrong vectors.

Question 10

(a) The majority of candidates correctly factorised the quadratic equation and found the correct values of $\tan\theta$. A few candidates mistakenly rejected the solution $\tan\theta = -1$, considering it invalid. However, very few candidates found all the correct angles within the specified range. Most found the angles 15.9° and −45°, as these were directly presented by their calculators. Many candidates recognised that there would be additional solutions within the range but made errors in identifying them. A common mistake was treating the tangent function as an even function and incorrectly offering −15.9° and 45° as alternative solutions. Some candidates also provided 164.1° as a solution. A few candidates rounded 15.9° to 16°, which led to the loss of a mark, particularly if 15.9° was not clearly shown in their response. Additionally, −164° was not accepted unless properly qualified.

(b) The vast majority of candidates gained the first mark by finding a value for $3\phi - 1.5$. While some candidates also found 2.41 radians, very few produced the third value in the range, 7.01. Many candidates then used the correct order of operations to find at least one value of ϕ, although many

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gave the first angle as 0.74 rather than 0.743 or better. Not giving answers to 3 significant figures or better meant that many candidates did not gain full marks even though their solutions were otherwise fully correct. Many rounded or truncated their work early, starting with $\sin(3\phi - 1.5) = 0.7$, which potentially led to errors in the final answer.

Question 11

(a) Most candidates were able to determine the common difference, although it was often expressed as $\log_x 27$. If this form was then used in the expression for the sum of the first n terms, the method mark was not awarded unless the common difference was eventually simplified to $3\log_x 3$, as the method needed to lead to the required form for the sum. Candidates who correctly expressed the common difference $3\log_x 3$ were generally more successful in completing the question.

Candidates should be aware that if the question specifies a particular form for the answer, the solution must be presented in that form. In this case, the answer had to be expressed as $k\log_x 3$ to earn full marks. Another common error, which led to the loss of the final answer mark, was presenting solutions without suitable brackets, even if the intention appeared to be correct.

(b) The vast majority of candidates identified that the common ratio was $3\tan^2\theta$. It was also clear that most candidates appreciated that the modulus of the common ratio had to be less than 1. However, many of those candidates who took the next step stated that $\tan\theta < \pm\frac{1}{\sqrt{3}}$ without rejecting the negative possibility. There are two points here; firstly, that expression is meaningless, or at best partially incorrect, and secondly, it does not lead to the correct answer such that, even where the correct answer was then given, only three out of the possible four marks were awarded. For those candidates who gave a final answer, many just gave $\theta < \frac{\pi}{6}$ rather than $0 < \theta < \frac{\pi}{6}$ and therefore did not gain the final mark.

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ADDITIONAL MATHEMATICS

Paper 0606/21
Paper 21

Key messages

Candidates should read each question carefully and identify any key words or phrases. For example:

- When an **exact** answer is required, it usually means that a surd, logarithm, exponential, π and/or a fraction is needed.
- When the instruction in the question is **Show that** …, all method steps needed to find the answer that has been given must clearly be stated, as no part of the method will usually be implied.
- When the answer must be stated in a **particular form**, candidates should check they have stated their final answer in that form.

It is important for candidates to know that, to be credited for work following an error, they must show their method. A correct method will not be assumed from an incorrect value if the method used to find it has not been seen.

Candidates should check that their calculator is in the appropriate mode when working with trigonometric expressions, particularly in questions involving calculus or the solving of trigonometric equations. Candidates should also be aware of how to use their calculator to check their solutions.

General comments

Using the most efficient method usually resulted in shorter and more accurate solutions. For example, in **Question 2(a)**, candidates who used the trigonometric identity $\sec^2 x = 1 + \tan^2 x$ to rewrite the derivative, produced neater and more accurate solutions than candidates who rewrote $y = \tan x - x$ in terms of $\sin x$ and $\cos x$.

When a question included the key word 'Hence...', candidates were expected to use a previously found result to answer the current part of the question. This indicated that a specific skill was being assessed. This was the case in **Question 2(b)**, for example.

Some candidates demonstrated good problem-solving skills. For example, in **Question 3(b)**, these candidates formed and carried out a multi-step plan which involved a combination of skills.

Poor presentation sometimes resulted in miscopying errors. Some candidates gave well-presented solutions. This made the logic of their method easy to follow. Other candidates needed to improve the presentation of their work. Presentation was often poor in **Question 4**, for example.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

This question proved to be an accessible start to the paper for most candidates and these candidates offered fully correct solutions. The majority of candidates chose the simpler initial step of writing $\tan\theta + \cot\theta$ in terms of $\sin\theta$ and $\cos\theta$. The few candidates who chose to rewrite $\tan\theta + \cot\theta$ in terms of $\tan\theta$ sometimes made slips in rearranging.

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Question 2

(a) This part of the question was generally well attempted with a good proportion stating the correct derivative. Many candidates offered a fully correct solution. In this question, the simplest approach was to find the derivative in terms of $\sec^2 x$ and then to use the trigonometric identity $\sec^2 x = 1 + \tan^2 x$ to rewrite the derivative. Those candidates who rewrote $y = \tan x - x$ in terms of $\sin x$ and $\cos x$ were less successful, as some were unable to differentiate the expression that resulted. A few candidates used θ instead of x which was condoned on this occasion.

(b) The majority of candidates were able to earn the first mark for a correct statement. Some candidates went on to complete the solution correctly, stating the required answer in exact form. A few candidates ignored the key word 'Hence' and made no use of **part (a)**. A small number of candidates did not substitute the limits correctly. Some other candidates did not evaluate the definite integral correctly, either offering an incorrect exact value, such as $\frac{3\pi}{4}$, or stating an inexact decimal answer.

Question 3

(a) Most candidates rewrote 8 as 2^3. Candidates who stated and used a substitution were more successful. Some candidates stated $x = 3$ without any evidence of method. This was not condoned for full credit. In weaker responses, candidates used an incorrect order of operations or attempted to take the logarithm of each term. Some other candidates attempted incorrect factorisation, with $8^{\frac{1}{x}}$ as a common factor. Presentation of responses was often poor and work was often difficult to follow and difficult to credit.

(b) Candidates found this part of the question to be challenging. Few candidates saw the need to equate constants and coefficients of $\sqrt{3}$. Those candidates who formed equations almost always solved them successfully to find the correct values. A good number of candidates earned partial credit for squaring the left-hand side, although occasionally the 3 was given as 9.

Question 4

The simplest approach was to form $q(\mathbf{b} - \mathbf{a}) = p(\mathbf{c} - \mathbf{b})$ and rearrange. This involved a simple change of subject process that naturally generated the denominator $p + q$ without too much effort. Candidates who tried other routes often struggled with the correct scalar using p or q rather than $\frac{p}{p+q}$, for example. A fair number of candidates earned partial credit for two correct statements. Responses were often poorly presented. In weaker responses, candidates often equated vectors to scalars.

Question 5

Candidates were more successful in this question and a good number of correct solutions were seen. Candidates who worked one step at a time made more progress than those who attempted to combine several steps. For example, those candidates who applied correct laws of logarithms to the terms that were in the same base as a first step and dealt with the change of base in the next step often offered better solutions. A good proportion of candidates were able to change the base of the second term correctly. Those few candidates who did not carry out a correct change of base, sometimes stated $\frac{1}{\log_a p}$ or even $-\log_a p$ after some incorrect manipulation. A few sign slips were made when forming the quadratic equation. Some candidates who had an incorrect equation did not show their method of solution and were therefore not credited for this step, as a correct method was not assumed from an incorrect value.

Question 6

(a) Most candidates were able to formulate a correct plan and a good proportion of fully correct solutions were seen. A small number of candidates did not offer a final answer in exact form. A few candidates worked in decimals from the start. These may have improved if they had given more attention to the key phrase in the question 'giving your answer in terms of π'. All candidates worked

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in radians, as expected. A few candidates were unable to find the correct value of r, whereas most candidates were able to find the arc length.

(b) Again, most candidates formed a correct plan. This was usually: area sector *AOEF* + area of triangles *ABO* and *EDO* + area of triangle *BCD.* Some candidates attempted area segment *AFE* + area rectangle *ABDE* + area of triangle *BCD*. A few candidates omitted to find the height of the triangle *BCD* and offered the area of triangle *BCD* as 15, from $\frac{1}{2} \times 6 \times 5$. A small number of candidates tried to find the angle *BCD* and use $\frac{1}{2}ab\sin C$ but were not successful as they commonly attempted to use $\frac{\pi}{3}$, half of angle *AOE*. A small number of candidates were unable to recall the formula for the area of a sector with the angle in radians, omitting $\frac{1}{2}$, for example.

Question 7

It was rare for a fully correct solution, with an exact answer, to be offered. A fair number of candidates could not be awarded full marks as they stated their final answer as a decimal. Some candidates had a negative area as they did not interpret the lengths of the sides of the triangle as all needing to be positive. A good number formed the correct equation of the normal, although occasionally the gradient form $\frac{x - x_1}{y - y_1} = -\frac{1}{m}$ was applied or candidates considered that the gradient was the gradient function of x rather than the numerical value of this when $x = \pi$. In the weakest responses, candidates made no attempt to use the product rule. Sometimes these candidates found the coordinates of a second point and found the gradient of a chord rather than the tangent. Some candidates would have improved if they had shown a clearer step-by-step solution, rather than combining several steps into one, with an error.

Question 8

(a) A good proportion of candidates found an expression for the velocity, formed a quadratic equation and solved it, discarding the negative solution. A small number did not reject the negative solution and may have benefitted from reading the question more carefully. Candidates who incorrectly differentiated and then stated a solution of their equation without any evidence of method were not credited. Some candidates found both the first and second derivative, equated the second derivative to 0 and attempted to solve this to find their value of t. A few candidates made no real attempt to answer.

(b) This part of the question was very challenging. A small number of candidates earned partial credit for finding both the displacement and the velocity when $t = 0$, but offered no conclusion using this information. A few candidates earned partial credit for finding either the displacement or the velocity when $t = 0$. Some of these candidates found the value of the displacement for two values of t and attempted to come to a decision using the values found, misinterpreting the question.

(c) Candidates needed to use the information they had found in **part (a)** in this part of the question. To find the total distance, they needed to correctly combine the displacements when $t = 0$, $t = \frac{1}{3}$ and $t = 2$. Very few candidates offered a fully correct solution. The most common error was to find $x_2 - x_0 = 18 - 8 = 10$ or just state $x_2 = 18$ as their answer. Other candidates integrated the velocity between the limits $t = 0$ and $t = 2$. Some other candidates attempted to integrate the displacement between the limits $t = 0$ and $t = 2$. A few candidates attempted to combine the displacements when $t = 0$, 1 and 2.

Question 9

(a) (i) This part of the question was well answered with almost all candidates stating the correct value.

(ii) Most candidates differentiated, found $x = 4$ and attempted to use this in some way. A few candidates correctly solved $3 = 4^2 - 8(4) + c$. It was necessary to form a correct equation in c to earn the second mark, not all candidates managed this. A small number of derivatives included a

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'+c' term. Some incorrect use of $\geqslant$, rather than =, was seen. An answer of $c \geqslant 19$ was not condoned.

In weaker responses, candidates attempted to use $3 = x^2 - 8x + c$ incorrectly. For example, candidates:

- solved $0 = x^2 - 8x + 3$ for x and stated the answers as values for c
- substituted $x = 0$ into $3 = x^2 - 8x + c$ and solved
- stated the discriminant of $x^2 - 8x + c = 0$ as being negative and solved for c.

(b) This was the most challenging part of the question for the majority of candidates. It was expected that candidates would either appreciate that, at the minimum point, $x = 4$ and solve $0 = 16 - 32 + c$ for the boundary value from which $c > 16$ could be deduced or would use the completed-square form to deduce that $c - 16$ needed to be positive. Very few candidates were able to offer a correct solution. Most candidates did not seem to have a correct plan.

Question 10

(a) A small number of correct answers were seen. Common errors were:

- to offer ${}^7C_3 \times {}^{10}C_1 \times {}^8C_2 = 9800$
- to use permutations instead of combinations
- to add where multiplication was appropriate
- to choose numbers of girls and boys independently of the sum needing to be 6.

(b) Candidates found this much more challenging and a fully correct solution was rare. A few candidates earned partial credit for finding 1080, omitting to take into account the number of possible positions for the number.

Question 11

(a) (i) Candidates struggled to offer an explanation that was valid and acceptable responses were rare. Some explanations were completely incorrect. Other explanations were not sufficiently clear to be credited. Some candidates simply described that a function took an input and generated an output, either generally or with a specific example.

(ii) A small number of candidates earned full credit in this part of the question. The remaining candidates formed the correct unsimplified expression but made a slip in the algebraic manipulation.

(iii) Some clear and correct statements were seen, although these were not common. It was more common for candidates to try to find the inverse function, make a slip and then state a relationship based on this work rather than the previous part of the question.

(iv) Candidates struggled to offer a valid explanation in this part of the question. The simplest explanation, involving the symmetry of the curve in the line $y = x$, was rarely seen. Many comments were just descriptions of the graph, for example, using the asymptotes or simply restating the idea that the function was self-inverse without reference to the graph.

(b) There were no correct responses to this part of the question. The most common incorrect answer was g $\geqslant$ 2. Sometimes candidates offered g $\leqslant$ 2 but 1 < g was never observed even though this could be read from the graph.

(c) This part of the question was better answered, with a good number of correct responses seen. Incorrect answers were, for example, $x > -\frac{1}{3}$, $x \geqslant 1$ and x is real.

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Question 12

(a) A good proportion of candidates earned full credit. Those who did not sometimes found the two values of *d* but were unable to progress, or used the sum formula rather than considering terms or, after finding expressions for the *n*th terms, omitted to form and solve an equation in *n*. A small number of candidates were unable to interpret the information given and made no real progress with their solution.

(b) Candidates who made no real progress with **part (a)** were unable to access this part of the question. A few candidates, who did make progress, either omitted the second step in the solution or omitted the final step. Some candidates made no attempt to answer.

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ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

Candidates should pay attention to the key words and phrases in each question. For example:

- When an **exact** answer is required, it usually means that a surd, logarithm, exponential, π and/or a fraction is needed.
- When the instruction in the question is **Show that** …, all method steps needed to find the answer that has been given must clearly be stated, as no part of the method will usually be implied.
- When the answer must be stated in a **particular form,** candidates should check they have stated their final answer in that form.

Candidates should also always show a logical and complete step-by-step method and not rely on their calculator to avoid the application of algebraic skills. Candidates should be aware that a calculator is useful as a checking tool, for solving equations for example, once they have stated their method of solution. It is also important for candidates to understand that some values that arise when solving equations are extraneous or spurious and are not valid for the original function. Candidates should, therefore, always check that the solutions they have found are valid by substituting them back into the original equation. Candidates should also ensure that, when solving trigonometric equations, their calculator is in the correct mode and the angles found are stated in the correct unit for the question.

It is important for candidates to know that, to be credited for work following an error, they must show their method. A correct method will not be assumed from an incorrect value if the method used to find it has not been seen. If candidates choose to use a substitution to simplify a solution, they should always state the substitution they have chosen before they use it.

General comments

The ability of candidates to correctly recall and apply specific techniques varied. For example, a good proportion of candidates were able to correctly recall the product rule for differentiation in **Question 2(a)** whereas fewer candidates were able to recall the correct small changes relationship $\frac{\delta y}{\delta x} \approx \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=\ldots}$ in **Question 2(b)**. Logarithms were often incorrectly interpreted or applied in this examination. This was evident in **Question 5(a)**, for example, where candidates often treated $\log_2 x^2$ as if it were $(\log_2 x)^2$. Candidates needed to take care to present their work in a mathematically correct form. In **Question 6**, for example, missing fraction bars and/or brackets were not condoned. This was particularly important in the step where candidates needed to rationalise the denominator. An expression such as $\frac{5-\sqrt{3}^{\;\times 8-4\sqrt{3}}}{8+4\sqrt{3}_{\;\times 8-4\sqrt{3}}}$ is not correctly expressed. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

This question combined the skills of solving simultaneous equations and working with indices. It was a reasonable start to the paper for a fair proportion of candidates. Some candidates stated $x = 0$, $y = 0$ as a solution. These candidates would probably have improved if they had checked that these values were solutions of the original equation. Some, but not all, candidates did reject spurious values. A good number of

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candidates earned a mark for correctly eliminating one unknown. Various methods of eliminating one unknown were seen. Those that used a simple form were usually more successful. For example, candidates starting with $\left(\frac{3}{2}x\right)^4 = \frac{27}{16}x^5$ perhaps made fewer manipulation errors than those who started with the form $\frac{\left(\frac{3}{2}x\right)^4}{x^5} = \frac{27}{16}$ or $\frac{y^4}{\left(\frac{2}{3}y\right)^5} = \frac{27}{16}$. A few candidates made arithmetic slips, even though a calculator was permitted. A few candidates made the solution more difficult by rewriting 27 and 16 as 3^3 and 2^4.

Checking the values found are valid in both equations is a sensible final step in the solution of simultaneous equations, but this was rarely seen. Candidates should be aware that, whilst using exact decimals is fine, the use of rounded decimals in the solving of simultaneous equations results in inaccurate answers and is not good practice. They are more likely to be successful if they use exact forms, such as fractions.

Question 2

(a) In this question, candidates could take one of two approaches. In the more popular method, candidates needed to rewrite the square root as a power and apply the chain rule and the product rule to find the required derivative. A good number of fully correct derivatives were seen. Some candidates went on to manipulate a correct derivative, some were successful, although not all. In this part of the question, it was not necessary to simplify the derivative and any manipulation was ignored once a correct statement had been seen. Some candidates omitted to multiply by 2 in their attempt at the chain rule. In weaker responses, candidates sometimes applied the chain rule but not the product rule. In other responses, candidates forgot the correct form of the product rule, and subtracted rather than added, or attempted the quotient rule. In the weakest responses, candidates took the square root term by term and then differentiated or were unable to subtract 1 from $\frac{1}{2}$ or added 1 to $\frac{1}{2}$.

The alternative approach was not commonly seen. In this approach, candidates attempted to rewrite the product as the square root of a single expression. It was not a particularly successful method as most omitted to square the x or were unable to successfully apply the chain rule to differentiate the expression that resulted.

(b) In this part of the question, candidates were expected to use the derivative they had found in **part (a)** in a correct small changes relationship. Some candidates offered fully correct solutions of acceptable accuracy. The best responses showed the substitution of $x = 4$ into their derivative and then substituted this into a correct small changes relationship. Candidates who started from the form $\frac{\delta y}{\delta x} \approx \left.\frac{dy}{dx}\right|_{x=4} \rightarrow \frac{0.06}{\delta x} \approx \frac{13}{3}$ were often more successful with the manipulation to the correct calculation for the change in x. A few candidates rounded their answer to fewer than 3 significant figures without stating a more accurate value, which was not condoned. These candidates possibly misinterpreted the key word 'estimate' here. The small changes relationship is already an approximate relationship. Candidates whose derivative was not correct needed to show the substitution of $x = 4$ to earn method marks. Not all candidates did this. The most common error, in otherwise reasonable solutions, was to start by writing $\delta x \approx \frac{dy}{dx} \times \delta y$. Some other candidates used 12.06 or 11.94 rather than 0.06 for the change in y. Weaker responses used $x = 12$ rather than $x = 4$. A few candidates made no attempt to answer.

(c) In this part of the question, candidates needed to find a stationary value. A reasonable number of fully correct responses were seen. Some candidates went on to find the value of the y-coordinate, which was not required. A few candidates made manipulation or transcription errors.

A little more care may have been needed in some cases as sign slips were sometimes made when solving $1 + 2x + x = 0$, for example. A few candidates found a spurious value using $(1 + 2x)^{-0.5} = 0$.

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Some candidates attempted to square both sides of $\frac{x}{\sqrt{1+2x}} = -\sqrt{1+2x}$ and then solve. These candidates were not always successful as those who used this route sometimes omitted to discard the spurious solution or were unable to manipulate the equation correctly. In some other responses, candidates made errors when dealing with the square root. Usually, these candidates took the square root of each term in the sum. If the derivative was not correct it needed to be of equivalent difficulty to the correct form for the method to be credited. In some responses, the manipulation in **part (a)** was so poor that the form that they were using was not of equivalent difficulty to that required. In the weakest responses, candidates attempted to solve $x\sqrt{1+2x} = 0$. A few candidates made no attempt to answer.

Question 3

(a) A good proportion of candidates offered fully correct responses to this part of the question. In order to be credited, candidates needed to form equations. Writing expressions alone was not sufficient. Some candidates did not state, or clearly imply, '= 0'. Equating expressions for p(2) and p(–1) was not enough to imply '= 0', as each expression could have been equal to any value. In some responses, candidates made sign errors when forming or solving the equations. For example, some candidates stated $a(-1^3) - 3(-1^2) - 3(-1) + b = 0$ and although some recovered from this to state a correct equation, a few did not. Some candidates confused roots and factors and attempted to use p(–2) = 0 and p(1) = 0.

(b) The simplest approach to this part of the question was to form the factors (x + 1) and (x – 2) and use these to deduce the missing linear factor (2x – 1), using the coefficient of x^3 and the constant. A few candidates used this approach but it was more common for candidates to form one of the linear factors and then attempt to find the quadratic factor that paired with it and factorise or use the quadratic formula to find the roots from it. A few candidates omitted to state all three roots in this part of the question. This was not condoned for full credit. Some algebraic slips were made in the factorising and some candidates who derived values other than x = 2 and x = –1 seemed to have forgotten the connection with **part (a)** by the time they had completed the solution to this part.

Question 4

Candidates needed to use a graphical method to solve the inequality. It was expected that this would be the intersection of two graphs. Methods that used a significant amount of algebra did not earn full credit, even if ultimately a graph was drawn from this algebra. Some candidates were able to form a correct plan using graphs. Very many more candidates resorted to solving using algebra in some way. The most common and obvious approach was to attempt to draw $y = |2x - 8|$ and $y = 4$. When candidates rearranged the inequality and used anything other than $y = |2x - 8|$ and $y = 4$, they needed to indicate the equation(s) they were using to be able to earn full marks. Graphs needed to be ruled and drawn accurately, not sketched, in order to solve using a graphical method. Not all candidates appreciated this. Those attempting to draw the graph of a quadratic function often offered inaccurate sketches. Some candidates stated their final solution set for $|2x-8| < 4$; perhaps rereading the question may have corrected this slip, although it may have been a misunderstanding of the initial inequality sign. A few candidates drew arrows to indicate sections of the graph that were relevant to the solution, but did not state the inequalities that were needed to represent these sections. Possibly they had misunderstood 'using a graphical method' as representing the solution set using a graph. Some responses were vertical lines at x = 2 and x = 6 after an algebraic process had been carried out. Again, it is possible that these candidates had misinterpreted 'using a graphical method' as meaning that they should represent the solution set graphically. In the weakest responses, graphs bore no relation to the inequality given, for example, some were of cubic curves.

Question 5

(a) In this part of the question, candidates who worked one step at a time were more successful than those who attempted to combine several steps. Some candidates earned a mark for one correct step applying knowledge of logarithms, such as bringing down the power in the first term, but made no further progress. A reasonable number of candidates also carried out a second correct step, such as a correct change of base. A fair number of these candidates went on to solve the equation correctly, writing the equation in exact exponential form and then solving. A small number of candidates used approximate values rather than keeping their values as, for example, a power of 2. This was not condoned for full credit. Some other candidates were unable to manipulate the

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equation correctly after a correct change of base. For example, when simplifying $\log_2 x^2 + \frac{\log_2 x}{4} = 18$, candidates choosing to multiply all terms by 4 often omitted to multiply $\log_2 x^2$ or, more commonly, 18 by 4. Many candidates treated $\log_2 x^2$ as if it were $(\log_2 x)^2$ and incorrectly formed and solved a quadratic equation.

(b) Candidates found this part of the question to be more challenging. In better responses, candidates stated and used a substitution such as $y = e^{2x+1}$ then formed and solved a simple quadratic equation in y. This was often completed successfully. A few candidates made sign slips when factorising an otherwise correct quadratic equation. A few candidates found a solution from $e^{2x+1} = -2$, often by incorrectly writing ln(–2) as –ln 2. In the weakest responses, candidates attempted completely invalid manipulation such as $e^{2x+1} - e^{10(-2x-1)} = 3$ or $\frac{e^{2x+1}}{10e^{-2x-1}} = 3$ or $-10e^{2x+1-2x+1} = 3$ or attempted to factorise with a common factor e^{2x+1} or took logarithms term by term and attempted to solve $\ln e^{2x+1} - \ln 10e^{-2x-1} = \ln 3$ by rewriting it as $2x + 1 - 10(-2x - 1) = 3$, or similar. Some candidates made no attempt to answer.

Question 6

Candidates could approach this question in a variety of ways but the most popular was to start with $\frac{5-\sqrt{3}}{(\sqrt{6}+\sqrt{2})^2}$, rewrite this as $\frac{5-\sqrt{3}}{8+4\sqrt{3}}$ and then rationalise the denominator. A fairly good number of fully correct solutions were seen. Some candidates needed to check their work more carefully as some arithmetic or sign slips were made. A few candidates needed to take more care when reading the question as $\sqrt{6}+\sqrt{2}$ was sometimes written as $6+\sqrt{2}$ or $\sqrt{6}+2$ at some point in the question. A small number of candidates wrote $\frac{5-\sqrt{3}}{(\sqrt{6}+\sqrt{2})^2}$ but then went on to attempt to rationalise as if they were simplifying $\frac{5-\sqrt{3}}{\sqrt{6}+\sqrt{2}}$. In weaker responses, candidates omitted working at key stages, which was not permitted as a calculator was not to be used and sufficient evidence of non-calculator working needed to be seen. The weakest responses started by writing $(5-\sqrt{3})(\sqrt{6}+\sqrt{2})^{-2}$ as $(5-\sqrt{3})\left(\left(6^{\frac{1}{2}}\right)^{-2} + \left(2^{\frac{1}{2}}\right)^{-2}\right)$ or applied an incorrect order of operations and attempted to expand $(5-\sqrt{3})(\sqrt{6}+\sqrt{2})^{-2}$ as if it were $(5-\sqrt{3})(\sqrt{6}+\sqrt{2})^{2}$, for example.

Question 7

(a) (i) A fair number of correct answers were seen. Common errors when seen were to offer: $^{10}P_5 = 30\,240$ or $10! = 3\,628\,800$ or an incorrect evaluation of $^{10}C_5$.

(ii) Candidates found this part of the question quite challenging. The simplest solution, $^8C_3 = 56$ was not commonly seen. Common method errors made were to: multiply $^8C_3$ by 2 or evaluate $^8P_3$ or $2 \times {}^8P_3$ or $^{10}C_3$.

(iii) Candidates found this part of the question to also be somewhat of a challenge. Various approaches could be taken. Perhaps the most obvious solutions, $^8C_4 \times 2$ or $^{10}C_5 - {}^8C_5 - {}^8C_3$, were not commonly seen. Common method errors made were to make one of the following evaluations: $2 \times {}^2C_1 \times {}^8C_4$ or $^{10}C_5 - {}^8C_5$ or $^{10}C_5 - {}^8C_3$ or $^8P_4$ or $2 \times {}^8P_4$ or $^9C_4$ or $2 \times {}^9C_4$.

(b) Very few candidates were able to offer any sensible plan in this part of the question. Correct answers were seen but were rare. A few candidates earned partial credit for finding 241 920. These candidates omitted to take into account that Abby and Ben could swop positions. A few other candidates earned a mark for considering 40 320 or 80 640. Commonly candidates drew diagrams which indicated correct positioning but they were unable to convert these diagrams into correct mathematics. Various incorrect attempts were made. For example, $3! \times 5!$ was commonly stated for one arrangement and $6 \times 3! \times 5! = 4320$ was a fairly common final answer.

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Question 8

Candidates needed to rewrite the given equation in terms of cosec 2θ or sin 2θ and rearrange to a form from which the quadratic equation could be solved using a standard method. A reasonable number of candidates did this and earned most, if not all, of the marks available. A few candidates omitted one or more possible solutions. Commonly the solutions omitted were the negative ones. Some candidates stated incorrect positive solutions in the range. A few candidates earned partial credit for a reasonable attempt to rewrite the given equation in terms of cosec 2θ or sin 2θ. Some of these made slips in rearranging to a solvable form. Some candidates stated and used a sensible substitution when solving. Other candidates correctly rearranged but made minor slips when solving the equation. Other candidates made method errors such as cosec 2θ(cosec2θ + 3) = 10 leading to cosec2θ = 10 and cosec 2θ + 3 = 10. A small number of candidates would have improved if they had reread the question before stating their answers as they gave their answers in radians which was not accepted. In weaker responses, candidates attempted to replace $\cot^2 2\theta$ with $\text{cosec}^2 2\theta$ or $\text{cosec}^2 2\theta + 1$. In the weakest attempts at a response, candidates attempted to replace cosec 2θ with $1 + \cot 2\theta$ or wrote, for example, $(\text{cosec}^2 - 1)2\theta$ or $\frac{\cos^2}{\sin^2}2\theta$. Some candidates made no real progress or made no attempt to answer.

Question 9

Candidates found the interpretation needed in both parts of this question to be somewhat challenging.

(a) In this part of the question, candidates needed to interpret an initial constant velocity of 6 ms^{-1} followed by a change of direction with a change in constant velocity. Some candidates were credited for the correct line from $t = 0$ to $t = 5$. A few candidates continued this line beyond $t = 5$ which was not acceptable. A few candidates drew the correct line, $v = -3$, from $t = 5$ to $t = 15$. Partial credit was given for a line satisfying some of the information given for the motion from $t = 5$ to $t = 15$, for example, a line at $v = -6$ or $v = 3$. In weak responses, candidates drew lines from (0, 0) to (5, 6) and $v = 6$ for $t = 5$ to $t = 15$, or drew other straight lines completely unrelated to the information given, or drew curves. Some candidates made no attempt to answer.

(b) In this part of the question, candidates needed to interpret a constant acceleration. For all the information in the question to be satisfied, candidates should have drawn a ruled, straight line passing through (0, –8), (10, 0) and (20, 8). Fully correct answers were seen but were not common. No scale was given on the vertical axis and candidates needed to indicate their own values. Some candidates found this difficult and either omitted to mark any scale at all or often made a sign error when marking –8. Some candidates were credited for a single straight line from (0, –8) to a point on the horizontal axis, usually (20, 0). In the weakest responses, candidates drew curves or several straight lines. Many candidates made no attempt to answer this part of the question.

Question 10

Although the techniques needed to produce correct solutions for all parts of this question were fairly routine, candidates often struggled to recognise the appropriate mathematical procedure for each given situation.

(a) In order to find the length required, candidates needed to find the coordinates of the maximum point on the curve. Various methods were acceptable for this but the most common, and the most successful, was to find the first derivative and use it to find the turning point. A few candidates found the x-coordinate of the turning point and added this to 4. Some candidates made arithmetic or sign slips and stated values that bore no relation to the diagram. A few candidates found values from incorrect working. This was not accepted. Some candidates unnecessarily used distance calculations to find the distance from (2, –4) to (2, 1) and these were not always accurately evaluated. Some candidates formed and solved $x^2 - 4x - 16 = 0$ in this part of the question. This was only accepted as a valid method if they went on to use the points of intersection they had found to find the turning point of the curve as (2, 1). Many responses to this part of the question were poorly presented with various attempts at different methods written haphazardly on the page, making solutions difficult to follow and difficult to credit.

(b) Candidates were more successful in this part of the question and a reasonable number earned full marks. A few candidates needed to read the question more carefully as they gave their answers to 3 significant figures without ever stating the exact form needed. Some candidates made sign slips when solving the equation using the formula or completing the square. This equation did not

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factorise and any attempt to factorise it was not credited. A few candidates made sign or arithmetic errors when writing the equation in a solvable form. Some of these candidates were able to earn a mark for showing the method of solution of their equation. Candidates who used their calculator to solve an incorrect equation without showing any method were not credited.

(c) This was the most challenging part of the question and one of the most challenging parts of the examination. Some candidates understood the need to substitute a sufficiently accurate *x*-coordinate of the point *A* into the first derivative of the curve. Only a few of these candidates were able to form and carry out a completely correct plan. A small number of candidates understood that the absolute value of the gradient of the line was the tangent of the angle they were looking for, given that the angle was acute. Some candidates attempted to find the equation of the tangent and use this in some way to find suitable lengths to use in a simple trigonometric calculation. Some of these candidates did this successfully. However, many of these candidates made slips when finding either the equation of the tangent or suitable lengths. This was not condoned as using this much less direct approach was unnecessary. In the weakest responses, when an answer was attempted, candidates found the angle made between the chord joining the maximum point to *A,* and the line $y = -4$. Many candidates made no attempt to answer.

Question 11

Candidates found the interpretation needed in all parts of this question to be somewhat challenging.

(a) In this part of the question, candidates needed to find and clearly identify the components of a velocity vector. The direction being evaluated could be implied in various ways, for example by stating the vector $\begin{pmatrix} 5 \\ 5\sqrt{3} \end{pmatrix}$. Using 'opposite = ' and 'adjacent = ', or writing values on the diagram alone, was not sufficient. Some candidates did not offer an exact form for the value of the *y*-component. These candidates may have improved if they had reread the question. Some candidates made no attempt to answer.

(b) Candidates now needed to use their answer from **part (a)** to form the velocity vector which then needed to be multiplied by *t* to become the position vector of the particle *A*. This needed to be correctly expressed as a vector to be credited. Some candidates did this successfully. Some candidates omitted to write their vector in terms of *t*, even though this was stated in the question, and stated only $\begin{pmatrix} 5 \\ 5\sqrt{3} \end{pmatrix}$ or similar. Other candidates offered a vector that was not correctly expressed, such as $\begin{pmatrix} 5\mathbf{i} \\ 5\sqrt{3}\mathbf{j} \end{pmatrix}t$. In the weakest responses, where an answer was attempted, candidates offered answers such as $\begin{pmatrix} 5 \\ 5\sqrt{3} \end{pmatrix} + 10t$ or $\begin{pmatrix} a \\ b \end{pmatrix} + t\begin{pmatrix} 5 \\ 5\sqrt{3} \end{pmatrix}$ where $\begin{pmatrix} a \\ b \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Many candidates made no attempt to answer.

(c) In this part of the question, candidates needed to form the velocity vector for particle *B* and use it to state the position vector in terms of *t*. Again, this needed to be correctly expressed as a vector to be credited. Some candidates were able to recognise this as being the appropriate mathematical procedure but very few were able to find the correct velocity vector. A small number of other candidates were able to simply write down the correct vector. A few candidates were awarded partial credit for one correct component or an attempt at a correct vector, such as $\begin{pmatrix} 2\sqrt{3} \\ 9 \end{pmatrix} + t\begin{pmatrix} 0 \\ \frac{5}{3} \end{pmatrix}$.

A very common error was to offer an answer of $\begin{pmatrix} 2\sqrt{3} \\ 9 \end{pmatrix} + t \times \frac{5}{3}$, which was not credited. It was rare for candidates to recover from this error and also state the correct vector. Other candidates offered a vector that was not correctly expressed, such as $\begin{pmatrix} 2\sqrt{3}\mathbf{i} \\ 9\mathbf{j} \end{pmatrix} + t\begin{pmatrix} \frac{5}{3}\mathbf{i} \\ 0 \end{pmatrix}$. In weaker responses,

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candidates attempted to find the unit vector in the direction of $\begin{pmatrix} 2\sqrt{3} \\ 9 \end{pmatrix}$ and use this in some way. For example, it was fairly common for candidates to either multiply it by $\frac{5}{3}$ and then use it as their velocity vector when forming their position vector or to use it directly as their velocity vector. Sometimes these candidates multiplied this unit vector by $\frac{5}{3}t$, or simply just t, and offered it as their answer. Many candidates made no attempt to answer.

(d) This part of the question was the most challenging of the examination. In order to show that the particles collided, candidates needed to use correct position vectors in some way. The simplest approach was to form and solve a pair of linear equations in t to show that the time was consistent. In this solution, candidates needed to show sufficient evidence of method to justify that the values of t were the same, as the wording of the question already suggested this would be the case. A few candidates were able to demonstrate this. A few other candidates were successful by solving one linear equation in t to find the correct time and then use this value of t to show that the position vectors were consistent at that value. Candidates who decided to use the fact that if the particles collided a value of t could be found from equating the distance between the two particles to 0, attempted a solution that was much more prone to errors. Whilst some of these candidates found a value of t from the quadratic equation they formed, some did not show the method of solution used. This was needed as the collision had to be fully justified. A few candidates demonstrated that the discriminant of the quadratic was equal to 0 so that it was possible the particles collided, but they did not show that the value of t was in fact a positive value. A few candidates earned partial credit for forming an equivalent equation using their vectors if they were of similar structure, so that the work was of equivalent difficulty. Very many candidates made no attempt to answer.

Question 12

A reasonable number of candidates formulated this problem into correct mathematical terms and selected and applied appropriate techniques to solve it. A good proportion of these candidates earned full credit. A few candidates found a value for x but not for h, the height of the cuboid. Some candidates made rounding errors, or used rounded values, that resulted in a slightly inaccurate value of h. Some candidates formed a plan that was correct except that they worked as if the cuboid were closed not open. Most candidates who made progress used $x^2h = 5$ to write h in terms of x and eliminate it from the expression for the surface area. Very few candidates wrote and used x in terms of h. Those candidates that did attempt this approach had a more difficult solution. A few candidates wrote $x^2h = 5$ but did not use it in any correct way. Some candidates made slips when simplifying the expression for the surface area. Other candidates made slips when differentiating $\frac{20}{x}$. Occasionally, candidates attempted to differentiate the expression for the surface area before they eliminated h. These candidates sometimes eliminated h from their expression for the derivative rather than from their expression for the surface area. This was an incorrect plan and could not gain full credit. In the weakest responses, candidates were unable to correctly form an expression in x for the volume of the cuboid. Some of these candidates stated $x^3 = 5$ and others used the formula for the volume of a cylinder, for example. Some candidates made no real attempt to answer.

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ADDITIONAL MATHEMATICS

Paper 0606/23
Paper 23

Key messages

- Candidates should read each question carefully and ensure that they have fully met the demands of the question.
- When a question involves the integration or differentiation of a trigonometric function, candidates should work in radians and not degrees.
- When a question asks for a particular method to be used, for example a graphical method or a non-calculator method, then candidates must ensure they use the required method.
- Candidates should consider carefully if two parts of a question are related and whether an earlier part of a question can be used to help them answer a later part.
- Candidates should consider the context of a question when deciding which values to include or reject.

General comments

Most candidates seemed to be suitably prepared for this examination and had sufficient time to attempt all questions. Some candidates produced high quality, clearly presented and logically sequenced responses which meant their mathematical reasoning was straightforward to follow and they were able to earn method marks following an arithmetic slip. Other candidates omitted steps from their working which meant at times they could not be awarded full marks for their response. This particularly applied to **Questions 3(a)**, **4(a), 9** and longer unstructured questions such as **Questions 5**, **8(b)** and **12(b)**.

Candidates should take care to re-write a number or symbol that they wish to change rather than trying to write over it, as this can be hard to read.

Candidates should be reminded to work in radians for any question where calculus techniques are applied to trigonometric functions, such as in **Question 8(b)** and **12**.

Comments on specific questions

Question 1

This question asked candidates to use the graph to solve an inequality and marks were not awarded for inequalities using over-accurate values from a calculator with no indication of the graph being used. Candidates who did well in this question drew the line $y = 1$ on the graph and used the x-coordinates of the points where this line intersected the curve to form two separate inequalities. Most candidates gave the x-values correct to 1 or 2 decimal places, and only a few sign slips were seen.

Candidates who made less progress on this question tried to solve the inequality algebraically, or used the points where the curve met the x-axis.

Question 2

(a) Many candidates were well prepared for writing a quadratic function in completed square form and used an efficient method to find the required form. Some slightly longer methods involved equating coefficients. Some candidates made a sign slip. This may have been avoided if they had checked their answer by expanding the brackets.

(b) Candidates who answered this part correctly understood that the maximum point of a quadratic function in the form $a - (x + b)^2$ has coordinates $(–b, a)$. Some candidates did not state the full

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range as they excluded ‘5’ and used $f < 5$. Other candidates may have improved their answer if they had considered the shape of the function as they wrote $f \geqslant 5$.

(c) Candidates who answered this part correctly realised that for g to have an inverse it needed to be a one-one function and correctly stated that $k = -2$. Some candidates incorrectly thought that finding the ‘least value of k’ meant they should present their answer as an inequality, rather than stating a single value.

(d) Responses to this part were very varied. Candidates who were successful recognised that they could rearrange the completed square form found in **part (a)** to find the inverse function. They then used their answers to **parts (b)** and **(c)** to find the domain and range of the inverse function, writing their answers using correct notation. Candidates who did less well omitted the domain and range, or did not relate the question back to the previous parts. Some candidates did not appreciate that an inverse function is one-one and incorrectly included the ‘$\pm$’ in front of the square root. Other candidates did not have a valid method for finding the inverse and solved the equation $g(x) = 0$ instead.

Question 3

(a) Many well-set-out and clearly-reasoned responses to this question were seen. A few candidates had difficulties in expanding the brackets or needlessly re-wrote the trigonometric functions in terms of $\sin\theta$ and $\cos\theta$. Candidates who produced the best answers appreciated that the left-hand side was in the difference of two squares form and then used the identity $\sec^2 A = 1 + \tan^2 A$, that is given on the formula page, to rewrite their expansion in terms of $\tan\theta$.

(b) Candidates were well prepared in solving trigonometric equations and most knew that ‘hence’ meant they should use the result in **part (a)**. Some candidates did not consider the negative square root when solving $\tan^2\theta = \frac{2}{3}$ and so did not find both possible values of θ.

Question 4

(a) Many candidates produced clearly set out and well-communicated answers to this part with the majority of candidates stating an equation for the area of the sector and then using this equation to eliminate the angle α in their equation for the perimeter. A few candidates did not complete the requirements of the question by stating the equation for the perimeter and left their answer as an expression. Candidates should always ensure they give their answers in the form requested in the question. A few candidates did not know the formulae for the area and arc length of a sector.

(b) Most candidates knew to differentiate the equation for P and then solve $\frac{dP}{dr} = 0$ in order to find the value of r corresponding to the stationary value. Candidates who did well in this question recognised that the context of the question meant they should reject the negative value of r. These candidates worked only with the positive value of r to find the value of P. They also correctly determined the nature of the stationary point, commonly using the second derivative test.

Candidates who did not relate their answer to the context of the question and gave a negative radius and perimeter could not obtain all the marks.

A number of candidates did not fully answer the question and either did not determine the nature of the stationary point, or did not determine the value of P at that point.

Question 5

Most candidates made a good start to this unstructured question and knew to use the quotient rule to differentiate the equation of the curve and find the gradient of the tangent at the point $x = 3$. Candidates who showed all of their working were able to be awarded method marks following any arithmetic slip. A few candidates found the correct equation of the tangent and made a slip when solving the equation of the tangent and the line $y = x - 16$ simultaneously. These candidates may have been able to correct their error if they had checked that the coordinates they had found were points both on the given line and the tangent. A

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few candidates incorrectly tried to find the intersection of the line and the curve and often only earned credit for finding the y-coordinate of the point on the curve at $x = 3$.

Question 6

(a) Many candidates integrated the function correctly and earned full marks. Successful candidates usually rewrote the integrand in the form $(3x+2)^{-\frac{1}{2}}$ and correctly identified that, on integration, the power would increase to $+\frac{1}{2}$. Some candidates multiplied by $\frac{3}{2}$ rather than divided by $\frac{3}{2}$.

Some candidates incorrectly thought that the integral should involve the natural logarithm.

(b) Many candidates were able to correctly integrate the exponential function and substitute the limits. Candidates who gave the best answers used correct notation, clearly showing they were integrating, and simplified the powers by using $e^0 = 1$.

Some poor notation was seen in this part and some candidates did not distinguish between integration signs and evaluation brackets. A few candidates reached the required answer and then tried to solve it as an equation. In the weakest responses, candidates attempted to change the power of the exponential function and commonly integrated as if the function were a power of x.

Question 7

(a) Many candidates correctly identified the 3rd and 6th terms and then equated them to find the correct value of x. Many well-set-out solutions were seen in this question.

Other candidates selected incorrect terms, possibly forgetting that the first term in the expansion is x^8. A few candidates did not use the laws of indices correctly when simplifying their terms. Some candidates did not know how to solve an equation involving x^3.

(b) (i) Answers to this part were more mixed. Some candidates did not realise that the 6th term would involve x^5 and that for the term to be a constant it would also have to involve $\left(\frac{2}{x}\right)^5$. Those who did usually correctly stated that $n = 10$. A common incorrect answer was $n = 12$.

(ii) Generally, candidates who identified that n was 10, were able to correctly find the 6th term in the expansion.

Question 8

(a) Many candidates were able to find both solutions to the equation and gave their answers in terms of π as requested. Some candidates only gave one solution. These candidates may have done better if they had re-read the question as it was clear from this that more than one answer was expected. A few candidates worked in degrees rather than radians.

(b) There was a wide variety in the quality of answers to this part. Candidates needed to formulate a correct plan. In better responses, the most common plan was to find the area between the curve $y = \sin 4x$ and the x-axis from $x = \frac{\pi}{24}$ to $x = \frac{5\pi}{24}$ and then to subtract the area of the appropriate rectangle under the line $y = \frac{1}{2}$. Candidates who did less well did not use their answer to **part (a)** as the limits for the integration and found it difficult to form a correct plan for finding the required area. Some candidates did not integrate $\sin 4x$ correctly or else tried to use limits in degrees. A few candidates did not give the area as an exact value and instead gave a decimal.

Question 9

A good number of fully correct solutions were given to this question. Candidates were not permitted to use a calculator in this question and so they needed to show a complete method in order to earn full marks. Good responses gave a clear rationalisation step and showed the expansion of the brackets and then a final

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simplification. Weaker responses made poor use of brackets, did not show a clear rationalisation step or showed insufficient evidence of working without use of a calculator.

Question 10

(a) Candidates who did well in this question recognised that they needed to find the value of n for which the sum of the geometric progression with first term 10 and a common ratio of 1.1 is greater than 200. These candidates set up a correct inequality for the sum, which they simplified and solved using logarithms. They then rounded the critical value up to the nearest integer. A few candidates did not round or else rounded down.

Many candidates misinterpreted the question and either tried to find the value of n where the nth term was more than 200, or else worked with an arithmetic progression.

(b) Many candidates were able to state two correct equations ($ar = a + 2d$ and $ar^2 = a + 6d$) connecting the equal terms in the arithmetic progression and the geometric progression. Only a few candidates were able to correctly solve these to find r. The most elegant solutions involved rewriting the equations as $a(r - 1) = 2d$ and $a(r^2 - 1) = 6d \Rightarrow a(r - 1)(r + 1) = 6d$ which divided to give $r + 1 = 3$, so $r = 2$.

Candidates who found $r = 2$ by assuming that, say, $a = 1$ usually only earned partial credit for an incomplete method. As a could be any value that satisfied the relationship $a = 2d$, candidates needed to establish this relationship as part of their solution.

Question 11

(a) (i) Many candidates found the correct number of orders by evaluating $3! \times 2!$ directly. A few found the total number of orders of 5 people and then subtracted the cases where 2 girls were together – this approach was often less successful as candidates did not consider all possible orders.

(ii) This part was found to be more challenging. Successful candidates usually found the total number of orders as 5! and then subtracted the number of orders where the boys were together by considering them as a single unit who could be sat AB or BA ($3! \times 2!$). Other candidates made little progress and did not show sufficient working. Some candidates did not consider the sense of their answer and gave answers that were greater than 5!.

(b) This question was found to be very challenging, and very few correct answers were seen. Many candidates still tried to use a permutations approach, but since the order within each group did not matter, the correct approach was to use combinations. Candidates who were successful considered 10 people being placed into 2 groups of 3 and 1 group of 4, so that Anjie and Bubay could each go into one of the first 2 groups. Candidates who tried to find the total number of ways and then subtract the number of ways where Anjie and Bubay were in the same group were rarely successful as they did not consider repetitions.

Question 12

(a) Most candidates knew to differentiate the velocity to find the acceleration and a high number of correct derivatives were seen. Many candidates knew to substitute $t = \frac{\pi}{3}$ into their derivative to find a, however some candidates did not have their calculator in 'radians' mode and so their evaluation was incorrect. Other candidates wrongly assumed that the acceleration should be positive and so gave their answer as 1.37 ms^{-2} rather than -1.37 ms^{-2}.

(b) This question differentiated well between candidates and some good responses were seen. These candidates found the first time at which the particle changed direction by finding when the velocity was first equal to 0 which was at $t = \frac{\pi}{4}$. They then went on to use either definite integration of v using the limits $t = \frac{\pi}{4}$ and $t = 0$, or they used indefinite integration and correctly found that the constant of integration was -1 before substituting $t = \frac{\pi}{4}$ into their equation for s.

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Candidates who produced less complete responses either made slips in their attempt to find the constant of integration, or assumed they should use $t = \frac{\pi}{3}$ from **part (a)**, or thought that the particle first changed direction when $a = 0$, rather than $v = 0$. Some candidates used degrees rather than radians and incorrectly found that the particle first changed direction after 45 seconds.

(c) Candidates found this final part challenging. Good responses compared the equations $a = -\cos t - \sin t$ and $s = \sin t + \cos t - 1$ and then compared directly to give $a = -s - 1$. Many candidates left this part blank or wrote $a = \frac{d^2s}{dt^2}$.

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